The GLEPS Package for Simulating Polarized Gamma Rays with GEANT3

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GLECS

GEANT3 Low Energy Compton Scattering

GLECS is an extension program for the GEANT3 simulation package that incorporates detailed physics into the simulation of Compton and Rayleigh scattering. In standard GEANT3, the free electron approximation is used for Compton scattering, and there are major errors in the way Rayleigh scattering is performed. These problems lead to significant errors in simulating scattering below a few hundred keV. The GLECS treatment of Compton scattering accounts for bound electron momentum on a shell-by-shell basis using evaluated data read from tables. For Rayleigh scattering, GLECS uses evaluated coherent scattering cross section and form factor data read from tables. Doppler broadening in Compton scattering can be switched on or off by the user.

available at:
http://public.lanl.gov/mkippen//actsim/glecs/

GLEPS

GEANT3 Low Energy Polarized Scattering

GLEPS is an extension of GLECS that includes the effects of polarized incident photons in the Compton and Rayleigh processes. The user specifies an initial polarization vector for each incident photon, which is then subsequently tracked during scatterings of the primary photon. The angular distribution of the scattered photon is based on the polarization vector of the photon at the point where it scatters. The resulting polarization of the scattered photon is also determined. Polarization effects are included only for the incident photon, and not for any secondary photons. GLEPS (like GLECS) can be used with practically any simulation program based on the GEANT3 (FORTRAN) system.

Also available at:
http://public.lanl.gov/mkippen//actsim/glecs/

Compton Scattering

The cross-section for Compton scattering depends not only on energy (ε) and Compton scatter angle (θ), but also on the azimuthal scatter angle (φ).

Photons tend to scatter at right angles to the incident polarization vector (φ = 90°).

$$\varepsilon = \frac{E}{E_o} = \frac{1}{1 + \frac{E}{mc^2}(1 - \cos\theta)}$$

$$\frac{d\sigma}{d\Omega} = S(x,Z)\left.\frac{d\sigma}{d\Omega}\right)_{KN} = S(x,Z)\frac{1}{2}r_o^2\varepsilon^2\left(\varepsilon + \frac{1}{\varepsilon} - 2\sin^2\theta\cos^2\varphi\right)$$

$$x = \frac{1}{\lambda(\text{Å})}\sin\left(\frac{\theta}{2}\right) = 0.0849 E_{keV}\sin\left(\frac{\theta}{2}\right) \quad \left[\text{in units of Å}^{-1}\right]$$

$$S(x,Z) \equiv \text{incoherent scattering function}$$

Rayleigh Scattering

Rayleigh scattering is the low-energy limit of Compton scattering in which the photon loses no energy (ε = 1).

$$\varepsilon = \frac{E}{E_o} = \frac{1}{1+\frac{E}{mc^2}\left(1-\cos\theta\right)} \xrightarrow[Rayleigh]{E\rightarrow E_o} \varepsilon = 1$$

$$\left.\frac{d\sigma}{d\Omega}\right)_{\text{Rayleigh}} = F^2(x,Z)\left(\frac{d\sigma}{d\Omega}\right)_{\text{Thompson}} = F^2(x,Z)\left(1-\sin^2\theta\cos^2\varphi\right)$$

$$F(x,Z) \equiv \text{atomic form factor}$$

Propagation of the Polarization Vector

The algorithm for determining the polarization vector of the scattered photon follows that of Namito and Hirayam (2000).

The direction of the resulting polarization vector depends on both θ and φ and is determined, in part, by the depolarization probabaility, 1-P, where,

$$1-P=\frac{\varepsilon+\varepsilon^{-1}-2}{\varepsilon+\varepsilon^{-1}-2\sin^2\theta\cos^2\varphi}$$

(Full details can be found in Namito and Hirayama.)

Polarization of Scattered Radiation

The polarization properties for both Compton and Rayleigh scattered photons are the same.

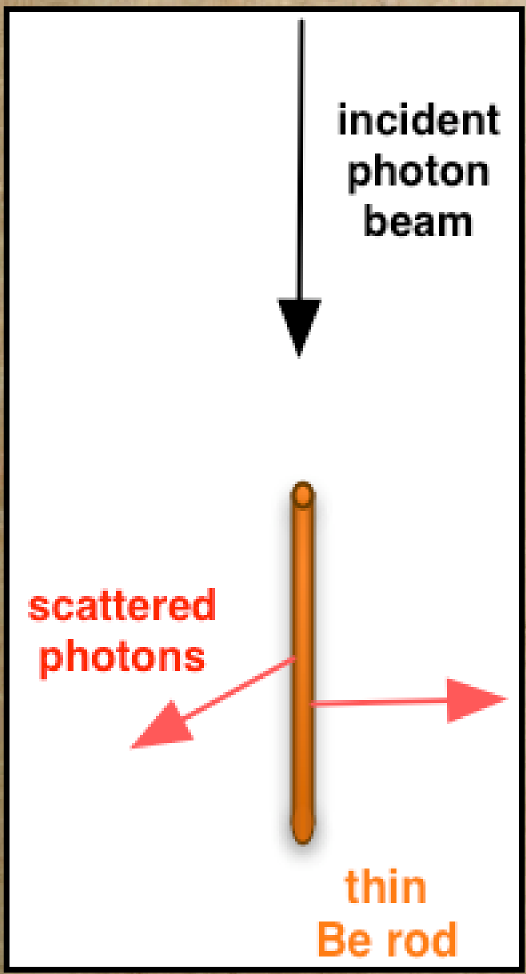
An initially unpolarized beam tends to become polarized when scattered, with a polarization level dependent on θ.

The polarization of an initially polarized beam depends not only on θ, but also on φ.

$$\Pi_{unpol}(\varepsilon,\theta) = \frac{\sin^2\theta}{\varepsilon + \varepsilon^{-1} - \sin^2\theta} \xrightarrow{\varepsilon\to 1} \frac{\sin^2\theta}{2 - \sin^2\theta}$$

$$\Pi_{pol}(\varepsilon,\theta,\varphi) = 2\left(\frac{1-\sin^2\theta\cos^2\varphi}{\varepsilon + \varepsilon^{-1} - 2\sin^2\theta\cos^2\varphi}\right) \xrightarrow{\varepsilon\to 1} 1.000$$

Validation Program



The validation results presented here use a simple scattering geometry.

A photon beam is directed down the long axis of a needle of material, in this case composed of Beryllium.

The direction of (singly) scattered photons is then recorded.

These validations have been performed using v1.24 of GLECS and v 3.21 14 of GEANT3 running under Mac OS X 10.3.5.

Total Cross Sections

The attenuation coefficients can be derived from simulations and compared with known values. Agreement is very good.

$I_o \equiv$ # of incident photons

$I \equiv$ # of transmitted photons

$I' \equiv$ # of scattered photons

x $\equiv$ length of Be rod

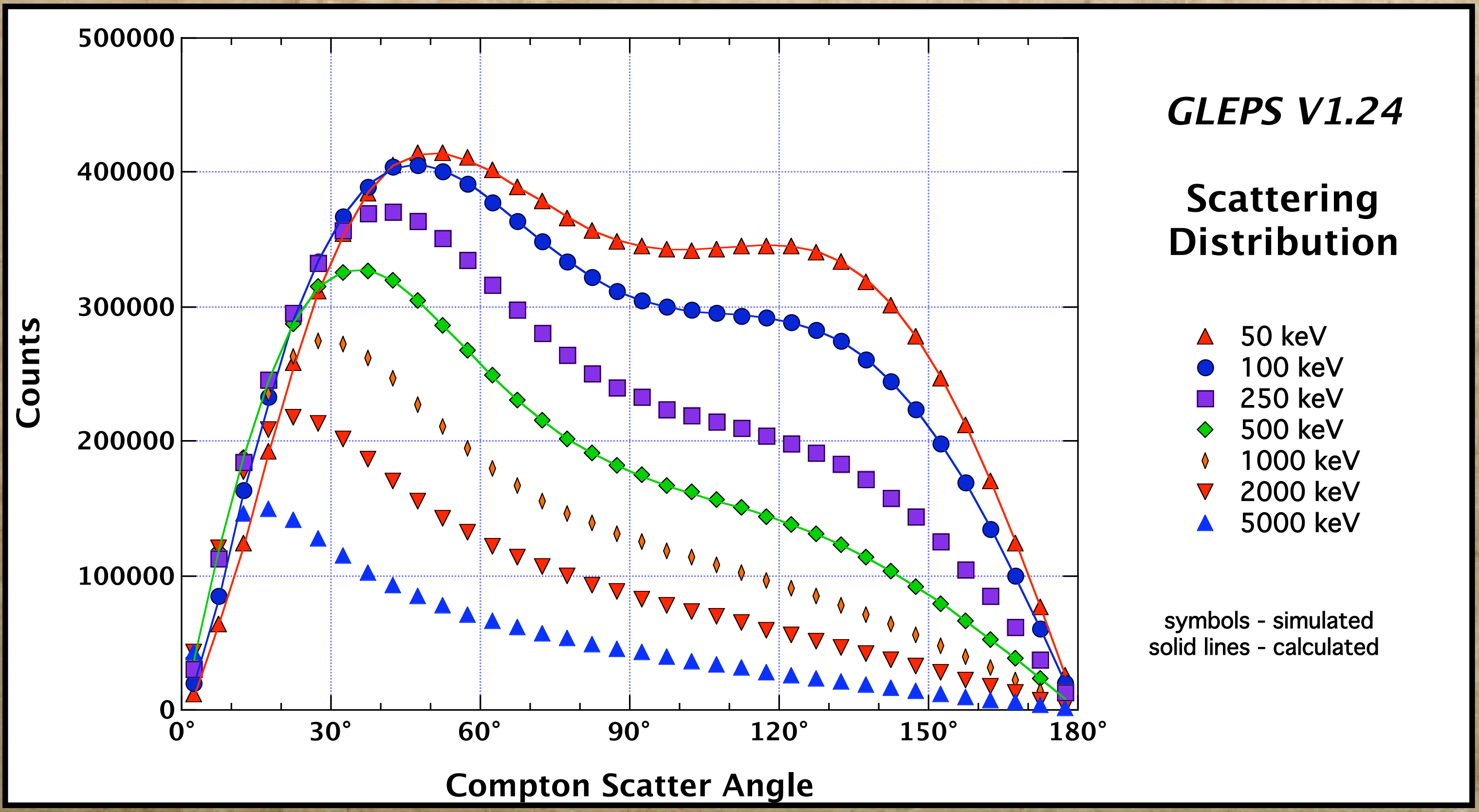
$$I = I_o e^{-\mu x}$$

$$-\mu x = \ln\left(\frac{I}{I_o}\right) = \ln\left(\frac{I_o - I'}{I_o}\right)$$

$$\mu = -\frac{1}{x}\ln\left(\frac{I_o - I'}{I_o}\right)$$

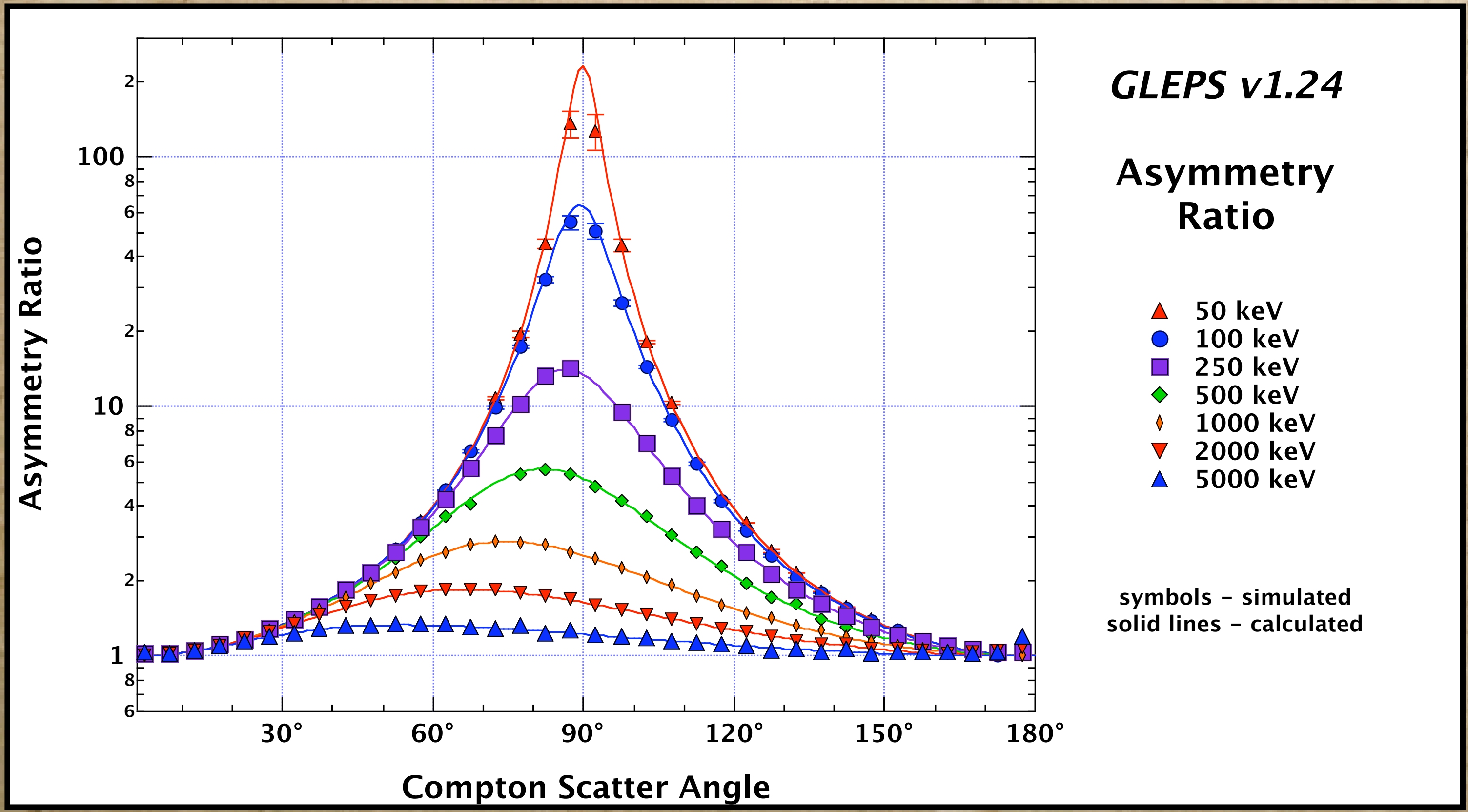
Energy (keV)	Incident Photons	Total Scattered Photons	μ Simulated	μ Calculated
50	25000000	10418298	0.2696	0.2717
100	25000000	9576280	0.2415	0.2421
250	25000000	7764199	0.1859	0.1870
500	25000000	6209403	0.1428	0.1429
1000	25000000	4707744	0.1043	0.1044

Compton Scattering Angular Distribution



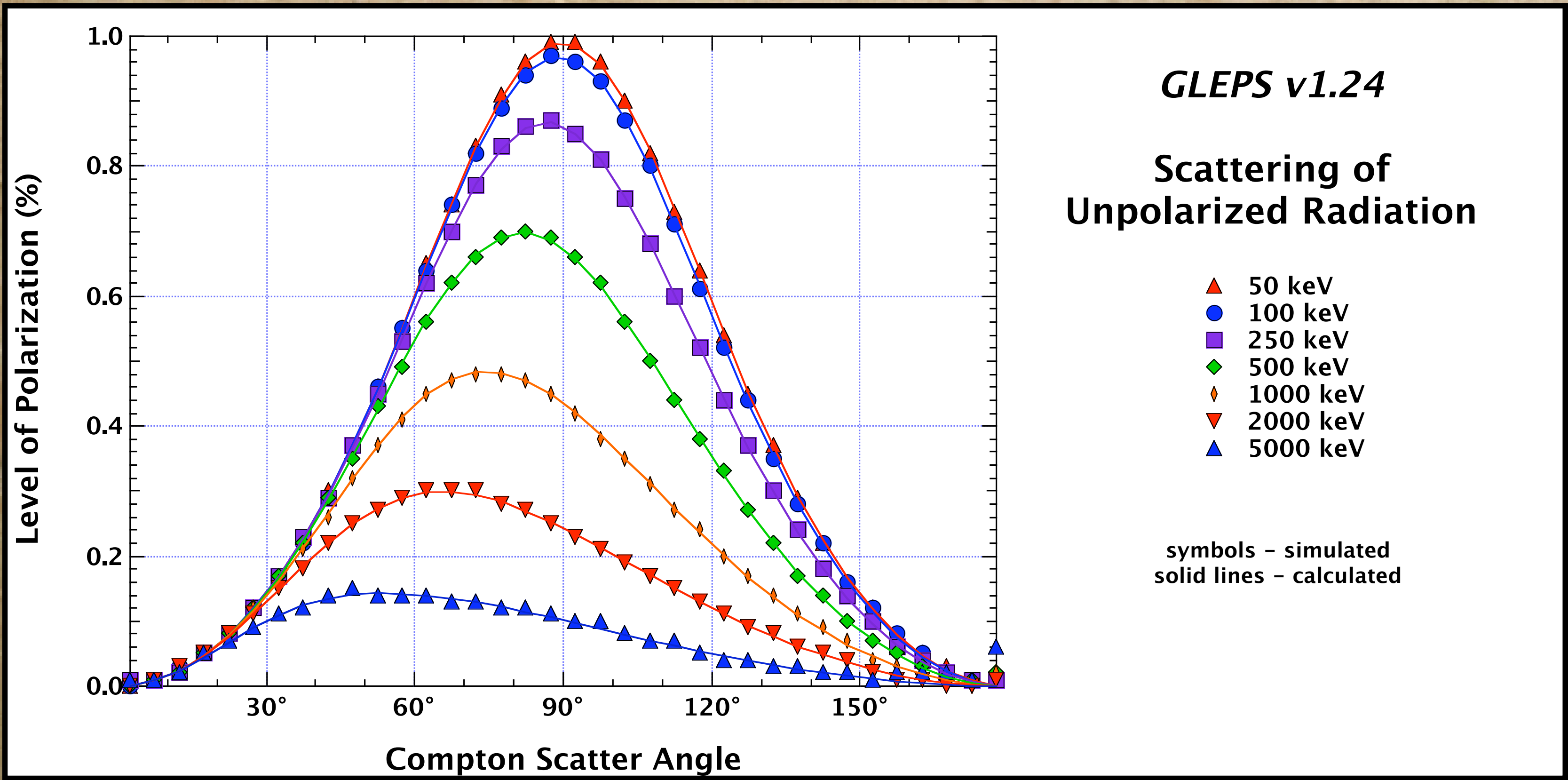
$$\frac{d\sigma}{d\Omega} = S(x,Z)\left.\frac{d\sigma}{d\Omega}\right)_{KN} = S(x,Z)\frac{1}{2}r_o^2\varepsilon^2\left(\varepsilon + \frac{1}{\varepsilon} - 2\sin^2\theta\cos^2\varphi\right)$$

Compton Scattering Asymmetry Ratio



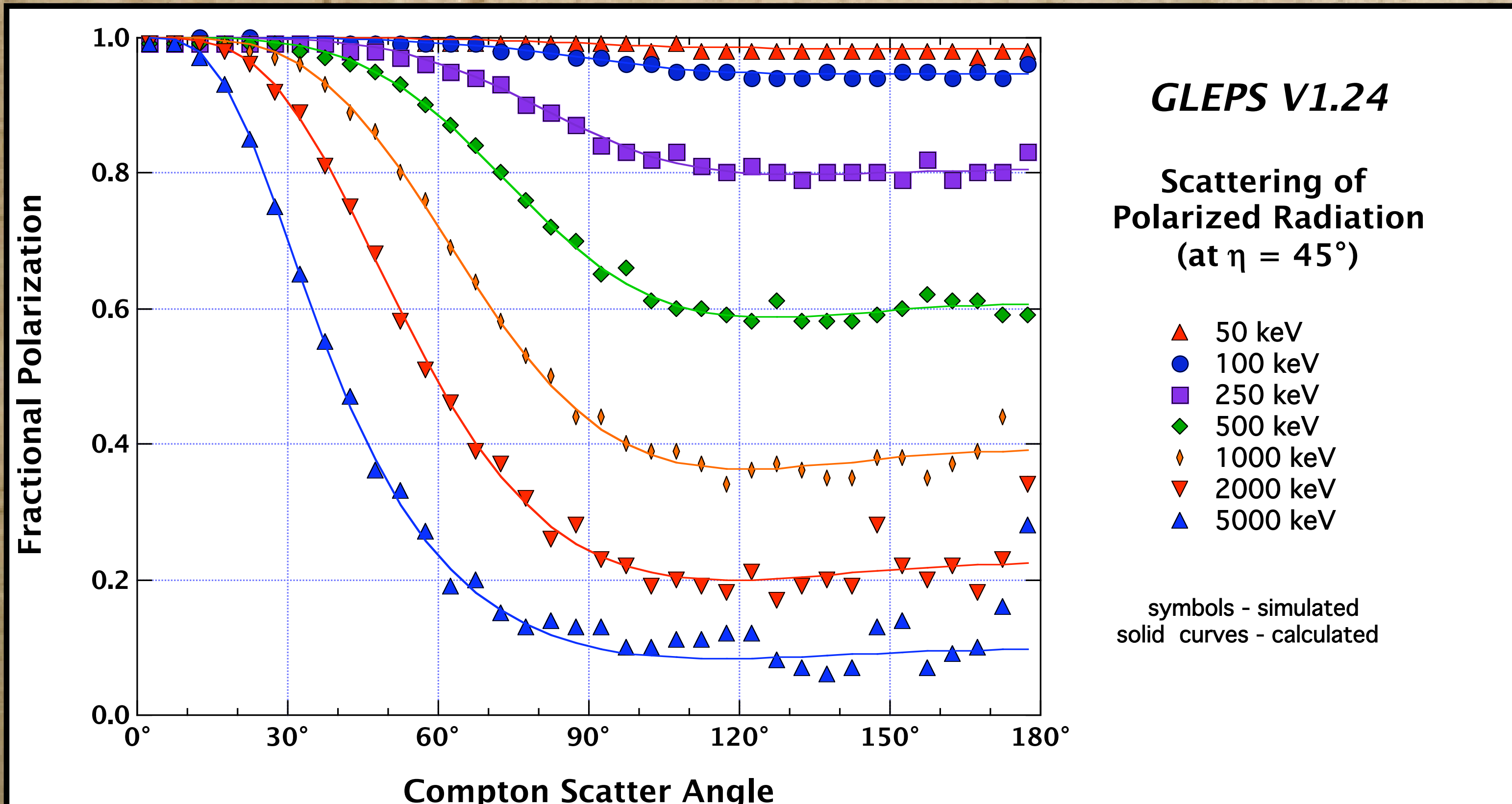
$$\frac{\frac{d\sigma}{d\Omega}(\varphi = 90°)}{\frac{d\sigma}{d\Omega}(\varphi = 0°)} = \frac{\varepsilon + \frac{1}{\varepsilon}}{\varepsilon + \frac{1}{\varepsilon} - 2\sin^2\theta}$$

Compton Scattering of Unpolarized Photons



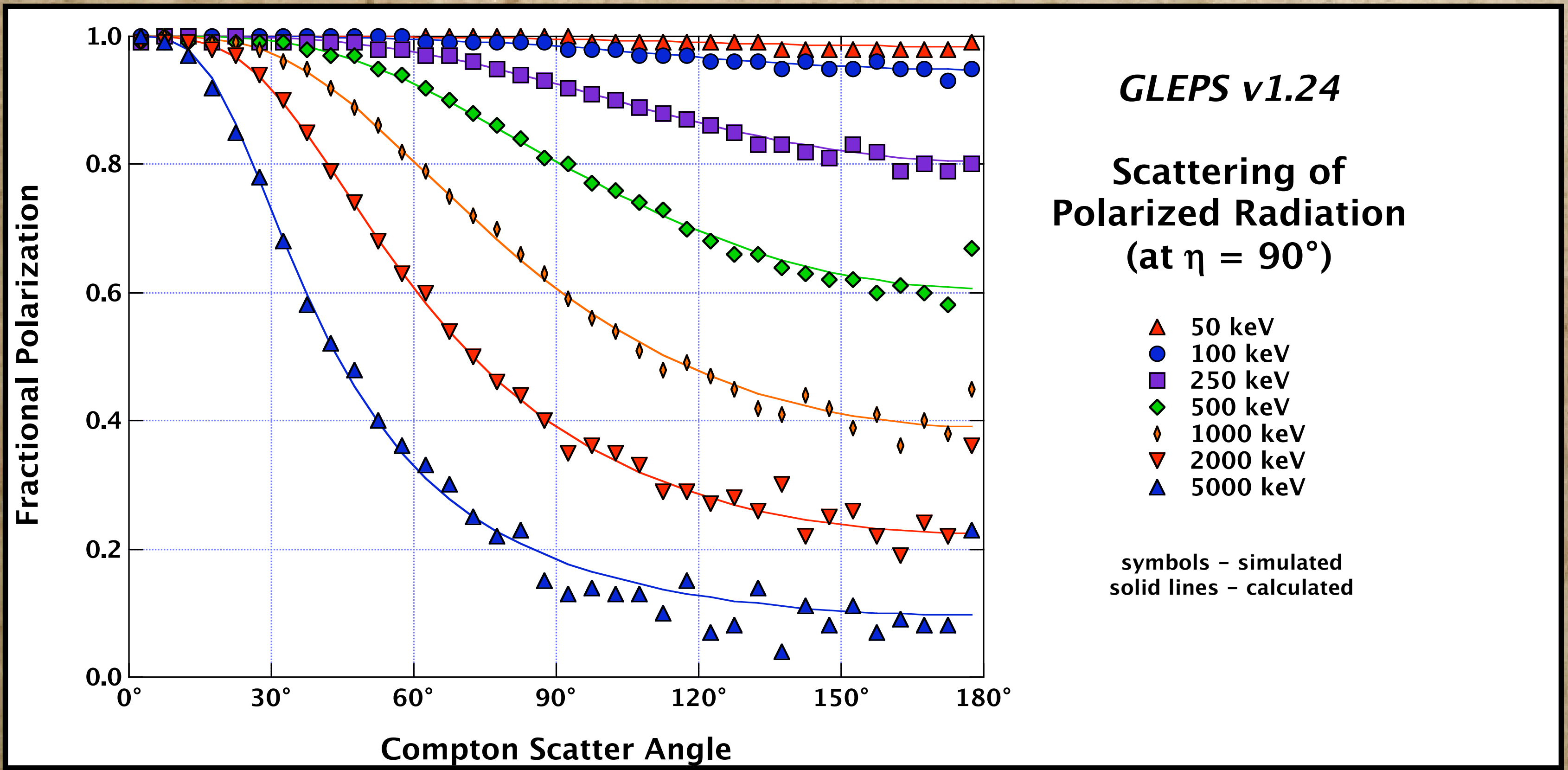
$$\Pi_{unpol}(\varepsilon,\theta) = \frac{\sin^2\theta}{\varepsilon + \varepsilon^{-1} - \sin^2\theta}$$

Compton Scattering of Polarized Photons



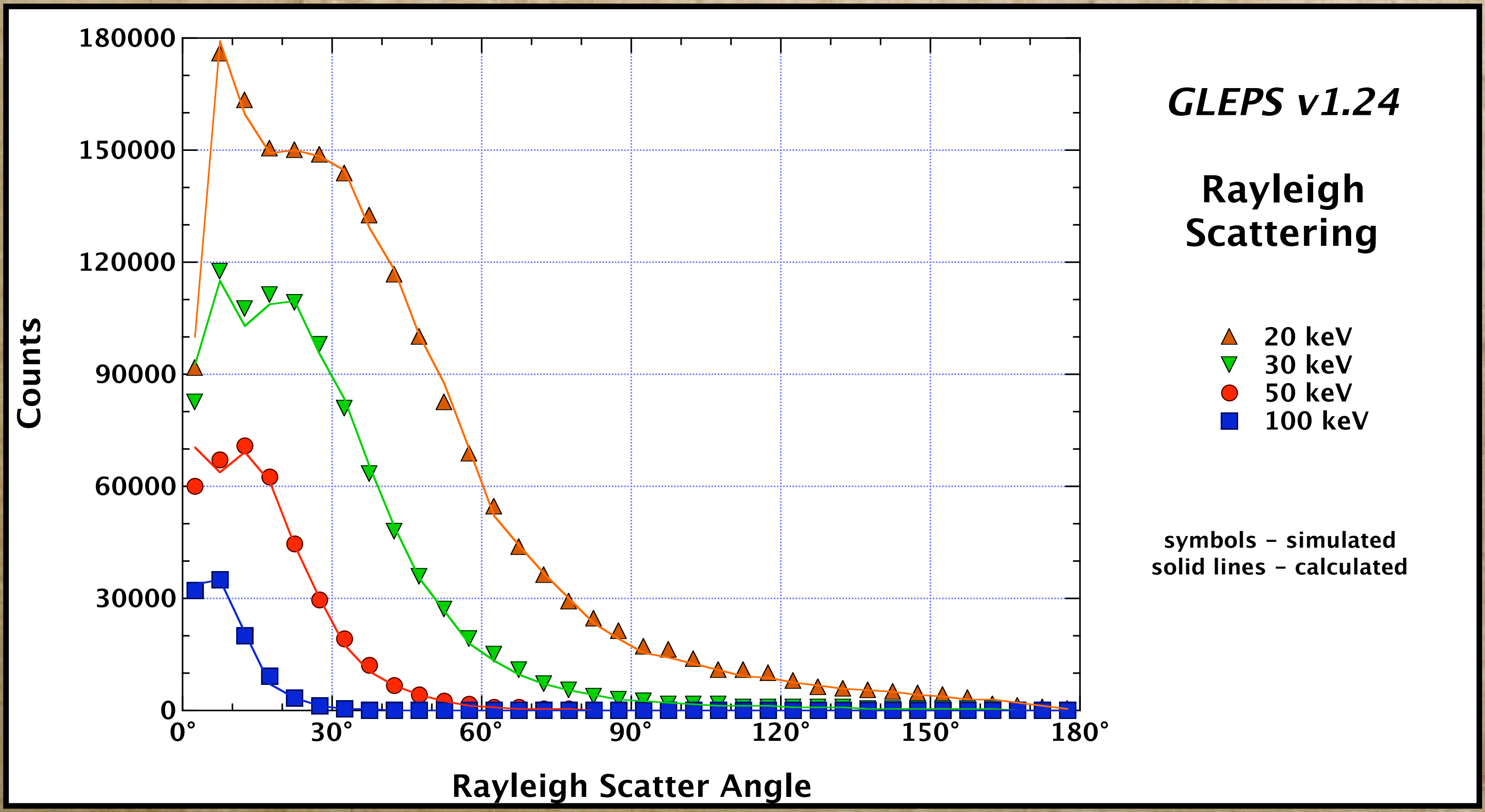
$$\Pi_{pol}(\theta,\varphi=45°)=2\left(\frac{1-\frac{1}{2}\sin^2\theta}{\varepsilon+\varepsilon^{-1}-\sin^2\theta}\right)$$

Compton Scattering of Polarized Photons



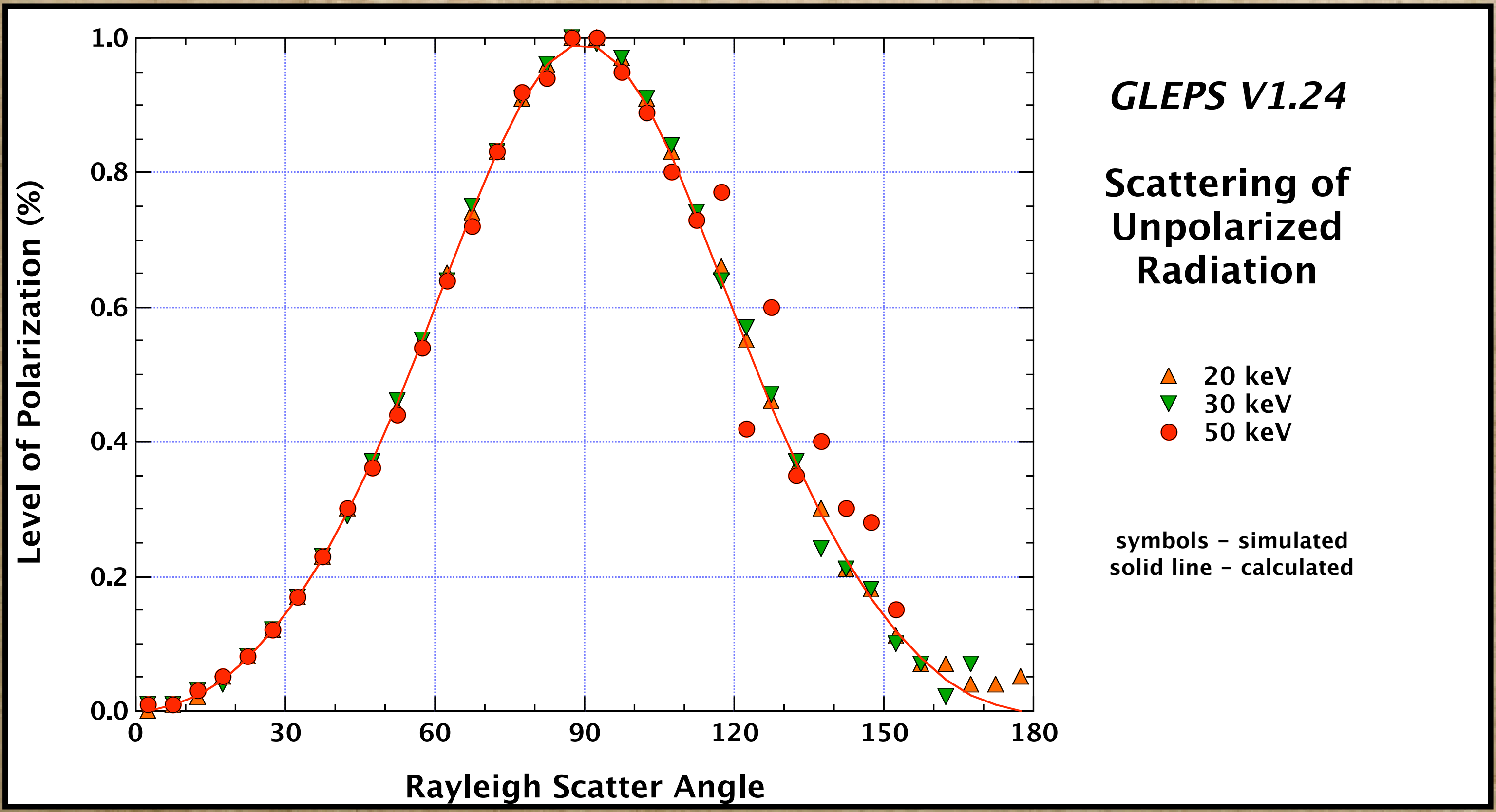
$$\Pi_{pol}(\theta,\varphi=90^\circ)=\frac{2}{\varepsilon+\varepsilon^{-1}}$$

Rayleigh Scattering Angular Distribution



$$\left.\frac{d\sigma}{d\Omega}\right)_{\text{Rayleigh}} = F^2(x,Z)\left(\frac{d\sigma}{d\Omega}\right)_{\text{Thompson}} = F^2(x,Z)\left(1-\sin^2\theta\cos^2\varphi\right)$$

Rayleigh Scattering of Unpolarized Photons



$$\Pi_{unpol}(\varepsilon,\theta)=\frac{\sin^2\theta}{\varepsilon+\varepsilon^{-1}-\sin^2\theta}\xrightarrow{\varepsilon\to 1}\frac{\sin^2\theta}{2-\sin^2\theta}$$

Summary

The GLEPS package has been developed to handle photon polarization effects in GEANT3.

Results presented here validate the physics, including the total cross-section, the angular distribution of scattered photons and the fractional polarization of the scattered photon beam.

The GLEPS package is currently being integrated into the MGEANT package, an implementation of GEANT3 designed for gamma-ray astronomy.

http://lheawww.gsfc.nasa.gov/docs/gamcosray/legr/mgeant/

References

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